New Physics and Neutrino Oscillation

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Abstract

Description of neutrino oscillation in the case of Non-Standard neutrino Interaction (NSI) is briefly presented. The NSI causes the entanglement between internal degrees of freedom of neutrinos (mass, spin, flavour) and other accompanying particles in the production and detection processes. In such case neutrinos are mostly in the mixed states. Role of the density matrix in description of neutrino oscillation process is shortly explained.

Theory of neutrino oscillation found in late sixties [1] works well and is almost commonly accepted. Now days two methods describing particles oscillation are considered, one based on the quantum mechanics (QM) and the second which use the quantum field theory (QFT) (for a review see [2]). For the standard relativistic neutrino interaction both methods are equivalent, and give the same result for final neutrino oscillation rate [3]. In the case of a NSI there is no, up to now, uniform way of describing the neutrino oscillation process. In the standard approach the state of produced neutrinos does not depend on the state of accompanying particles, states are disentangled. In such a case neutrinos oscillate in the way independent of the spins arrangement of the other particles present in the production and detection process. In the QFT approach, and in most studies of the neutrino oscillation beyond the Standard Model (SM) [4], oscillation process is assumed to be universal, where probability of neutrino transition is possible to be defined and depends only on the initial and final states.

However, beyond the SM the production and detection neutrino states may differ from states which are used to construct the Lagrangian of the model before the spontaneous symmetry breaking, the so called flavour states [5]. It can happen also that the right-handed neutrino fields appear, although there are a fairly strong limits on couplings involving these fields [6]. This causes that neutrino states in the production, detection and propagation processes need not to be given by the simple combination of neutrino mass states with the same helicity,

$$|\nu_\alpha, \downarrow\rangle = \sum_i U^*_{\alpha,i}|\nu_i, \downarrow\rangle, \quad |\overline{\nu}_\alpha, \uparrow\rangle = \sum_i U_{\alpha,i}|\overline{\nu}_i, \uparrow\rangle, \tag{1}$$

which is the basic assumption of QM description of the standard neutrino oscillation. In most approaches beyond the SM, instead of the states (1) the new one, separate for production (p)

and detection (d) are defined

$$|\nu_\alpha^p\rangle = \sum_i U_{\alpha,i}^p |\nu_i\rangle, \quad |\nu_\beta^d\rangle = \sum_i U_{\beta,i}^d |\nu_i\rangle, \tag{2}$$

where the mixing matrices $U_{\alpha,i}^p$ and $U_{\beta,i}^d$ are connected with the NSI Hamiltonians for production and detection

$$|U_{\alpha,i}^p|^2 \propto |\langle \nu_i; f_P |H^P| l_\alpha; i_P\rangle|^2, \quad |U_{\beta,i}^d|^2 \propto |\langle l_\beta; f_D |H^D| \nu_i; i_D\rangle|^2. \tag{3}$$

The $i_{P(D)}$ and $f_{P(D)}$ are initial and final states for production (detection) process. Then the amplitude of neutrino transition from the initial state $|i\rangle$ to the final state $|f\rangle$ (given by Eqs. (1) or (2)) is defined in the standard way

$$A_{f,i} = \langle f|e^{-iHt}|i\rangle, \tag{4}$$

where H is Hamiltonian of free neutrinos (in vacuum) or describing neutrino interaction with matter particles (in matter). In this description of the oscillation phenomena there is no possibility to take into account any kind of intrinsic correlation between neutrinos and other accompanying particles, but such internal correlations exist in some models of NSI.

The natural description of the oscillation phenomena is the QFT approach, where the production and detection neutrino states are not used, all calculations are done in the neutrino mass base and it is not necessary to assume that oscillations are described by production and detection independent probabilities [7]. In this description there is however one not very natural assumption that neutrinos propagate as virtual particles over macroscopic or even astronomical distance. Therefore another way of describing the oscillations, without the propagation of virtual neutrinos, where the QFT is used to construct the neutrino production and detection states only, has been proposed [8]. Taking into account that the QM (internal wave packets) and QFT (external wave packets) descriptions of neutrino oscillations are equivalent we proposed the generalization of both approaches where any kind of NSI can be taken into account [5]. The proposed description can be summarized in the following points:

(i) If the $M_{i,\lambda}^{P,\alpha}([\overline{p}],[\overline{\lambda}])$ are the amplitudes for "α" neutrino produced in the mass "i" and helicity "λ" state in the process: $A \to B + \overline{l}_\alpha + \nu_i(\lambda)$, and the $M_{k,\mu}^{D,\beta}([\overline{q}][\overline{\mu}])$ describe the "β" neutrino with mass "k" and helicity "μ" in the detection process $\nu_k(\mu) + C \to l_\beta + D$ then the amplitude for the $\alpha \to \beta$ transition after time T and at a distance L is given by [3]

$$iA_{\alpha\beta}(\lambda, \mu; [\overline{p}], [\overline{\lambda}], [\overline{q}], [\overline{\mu}]) = \tag{5}$$

$$\Theta(T) \sum_{k=1,2,3} M_{k,\lambda}^{P,\alpha}([\overline{p}],[\overline{\lambda}]) M_{k,\mu}^{D,\beta}([\overline{q}],[\overline{\mu}]) \int \frac{d^3p}{(2\pi)^3} F_{P,k}(E_k(\vec{p}),\vec{p}) F_{D,k}(E_k(\vec{p}),\vec{p}) e^{-iE_k(\vec{p})T+i\vec{p}\vec{L}},$$

where the $[\overline{p}]$ and $[\overline{\lambda}]$ $\{[\overline{q}]$ and $[\overline{\mu}]\}$ describe the average momenta and helicities of all other particles in the production (A, B, l_α) {detection (C, D, l_β)} process. The functions $F_{P,k}(E_k(\vec{p}),\vec{p})$ and $F_{D,k}(E_k(\vec{p}),\vec{p})$ are constructed from momenta distribution functions for all particles in the production and detection processes respectively without neutrinos (for details see [3]) and

$\Theta(T)$ is the Heaviside step function.

(ii) The probability for the overall process - production, oscillation and detection - is calculated as the modulus square of the amplitude (5) with proper averaging over initial and sum over final particles helicities and averaging over the necessary particles momenta in the same Lorentz frame (usually the rest frame of a detector). Practically, it is more convenient to calculate the production and the detection processes separately in their own rest frames, and in the final oscillation rates to distinguish the initial neutrino state and the final cross-section for the detection process.

(ii(1)) In the rest frame of A the neutrino initial state is calculated from dynamics of the production process $A \rightarrow B + \bar{l}_\alpha + \nu_i(\lambda)$ in the well know way. The neutrino density matrix in the mass (i) and helicity (λ) base $|i, \lambda\rangle$ is equal

$$\varrho^\alpha_{\lambda,i;\eta,k}(E, \theta, \varphi) = \tag{6}$$

$$\frac{1}{N_\alpha} \sum_{\lambda_A, \lambda_{A'}, \lambda_B, \lambda_l} \int \overline{dLips} M^{P,\alpha}_{i,\lambda}(\lambda_A; \lambda_B, \lambda_l; E, \theta, \varphi) \varrho_{\lambda_A, \lambda_{A'}} M^{P,\alpha*}_{k,\eta}(\lambda_{A'}; \lambda_B, \lambda_l; E, \theta, \varphi),$$

where the integral $\overline{dLips}$ is taken over the part of the phase space, without neutrino energy (E) and its momentum direction (θ, φ), the $\varrho_{\lambda_A, \lambda'_A}$ describes the polarization density matrix of decaying particle (A) and the factor N_α normalizes the density matrix, such that $Tr\varrho = 1$. It is convenient to normalize separately the initial state and the final cross section, then for the plane wave (not wave packet) approach the final oscillation rate is already normalized properly.

(ii(2)) To get the density matrix in the LAB frame, where decaying particles are moving (pions, decaying nuclei, muons), the Lorentz transformation, equivalent to the Wigner rotation, $\varrho_{LAB} = D^{1/2}_W \varrho_{CM} D^{1/2\dagger}_W$ has to be performed. For relativistic neutrinos in the helicity base, the matrix $D^{1/2}_W \simeq 1$, then the ρ_{LAB} to a very good approximation is equal to ϱ^{CM} [5]

$$\varrho^\alpha_{LAB}(E_b, \theta_b, \varphi) = D^{1/2}_W \varrho^\alpha_{CM}(E, \theta, \varphi) D^{1/2\dagger}_W \approx \varrho^\alpha_{CM} \equiv \varrho^\alpha(E, \theta, \varphi), \tag{7}$$

where the neutrino energy (E) and its spherical angle (θ) are expressed in terms of the appropriate quantities E_b and θ_b in the LAB frame after the Lorentz transformation.

(ii(3)) As neutrino detector is usually far away from the production place, to all practical purpose it is possible to use average density matrix defined in the following way

$$\overline{\varrho}^\alpha(E_b) = \frac{1}{\frac{d\Gamma(\Delta_b)}{dE_b}} \int_{\Delta_b} d\Omega_b \varrho^\alpha(\theta_b, \varphi) \frac{d^3\Gamma}{dE_b d\Omega_b}, \quad \frac{d\Gamma(\Delta_b)}{dE_b} = \int_{\Delta_b} d\Omega_b \frac{d^3\Gamma}{dE_b d\Omega_b}, \tag{8}$$

where Δ_b is the solid angle in which neutrinos "see" the detector. The $\frac{d^3\Gamma}{dE_b d\Omega_b}$ is the neutrino energy and angular distribution. For large baseline L, practically $\overline{\varrho}^\alpha(E_b) \simeq \varrho^\alpha(\theta_b = 0, \varphi)$.

(iii) The detection cross section is calculated in the standard way. Such a cross section describes the neutrino oscillation and detection process in one formula. Neglecting the coherence effect connected with the particles wave packets, the oscillation cross section is given

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by

$$\sigma_{\alpha \to \beta}(E_b, L) = \tag{9}$$

$$\frac{1}{32\pi s} \frac{|\vec{p}_f|}{|\vec{p}_i|} \frac{1}{2s_C + 1} \sum_{i,k;\lambda,\eta;[\overline{\mu}]} \int dLips \; M_{i,\lambda}^{D,\beta}(\vec{p}_f, [\overline{\mu}]) \; \overline{\varrho}_{i,\lambda;k,\eta}^{\alpha}(E_b) \, e^{i \frac{(m_k^2 - m_i^2)L}{2E_b}} \; M_{k,\eta}^{D,\beta *}(\vec{p}_f, [\overline{\mu}]).$$

The two formulae (6) and (9) are crucial for our analysis. From (6), calculating $Tr((\varrho^\alpha)^2)$, we see when the neutrino state is pure, or when it have to be considered as mixed. For example we can easy find that in the SM,

$$(\varrho_{\lambda,i;\eta,k}^\alpha)^{SM} = \delta_{\lambda,\eta}\delta_{\lambda,-1}U_{\alpha,i}U_{\alpha,k}^* , \tag{10}$$

which describes the pure quantum mechanical state. However, there are models of NSI in which neutrinos with both helicities can be produced. In such models, in the obvious way, the neutrino production states are mixed. But even if only the relativistic neutrinos with negative helicities are produced the states can be mixed too, it depends on the specific structure of the helicity amplitudes (see e.g. [9]). From the Eq. (9) we can easy find when the final oscillation rate factorize and when not.

In conclusion, we have presented the theory of neutrino oscillation applicable in the case of any NSI, where the internal degrees of freedom of neutrinos and other accompanying particles in a production and/or in a detection process are entangled.

References

[1] B. Pontecorvo, Sov. Phys. JETP, **26** (1968) 984; V. N. Gribov, B. Pontecorvo, Phys. Lett. **B28** (1969) 493.

[2] M. Beuthe, Phys. Rept. **375** (2003) 105.

[3] E. Kh. Akhmedov, J. Kopp, JHEP **04** (2010) 008.

[4] Y. Grossman, Phys. Lett. **B359** (1995) 141.

[5] M. Ochman, R. Szafron, M. Zralek, J. Phys., **G35** (2008) 065003; R. Szafron, M. Zralek, Prog. Part. Nucl. Phys., **64** (2010) 210.

[6] F. del Aguila, J. de Blas , R. Szafron, J. Wudka, M. Zralek, Phys. Lett. **B683** (2010) 282; F. del Aguila, J. A. Aguilar-Saavedra, J. de Blas, arXiv:1012.1327 [hep-ph]

[7] E. K. Akhmedov, A. Y. Smirnov, Phys. Atom. Nucl. **72** (2009) 1363.

[8] C. Giunti, IHEP **0211** (2002) 017.

[9] R. Szafron, M. Zralek, arXiv:1010.6034 [hep-ph].